Systematic Handling of Heterogeneous Geometric Primitives in Graph-SLAM Optimization

Supplementary Material

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Supplementary Material

Irvin Aloise, Bartolomeo Della Corte, Federico Nardi and Giorgio Grisetti
Department of Computer, Control and Management Engineering
Sapienza University of Rome
Email: {ialoise,dellacorte,fnardi,grisetti}@diag.uniroma1.it

JACOBIANS' COMPUTATION

In this additional materials, we provide the mathematical derivation of the Jacobians shown in the manuscript. In the remaining we indicate with $\mathbf{Z}_{ij}$ the factor directed from pose $\mathbf{X}_i = [\mathbf{R}_i \mid \mathbf{t}_i]$ to matchable $\mathbf{M}_j$. In this scenario, $\mathbf{\Delta X}_i \in \mathbb{R}^6$ and $\mathbf{\Delta X}_j \in \mathbb{R}^5$ will refer respectively to a pose and a matchable perturbation. We refer the reader to Sec. III and Sec. IV of the manuscript for details on the aforementioned objects.

Given this, the complete Jacobian deriving from factor $\mathbf{Z}_{ij}$ will be computed as follows:

$$\begin{aligned}\frac{\partial \mathbf{e}_{ij}(\mathbf{X} \boxplus \mathbf{\Delta x})}{\partial \mathbf{\Delta x}} &= \frac{\partial \mathbf{e}_{ij}(\mathbf{X}_i \boxplus \mathbf{\Delta x}_i, \mathbf{M}_j \boxplus \mathbf{\Delta x}_j)}{\partial \mathbf{\Delta x}} \\ &= \begin{pmatrix} \mathbf{0}_{7\times6} & \cdots & \mathbf{0}_{7\times6} & \mathbf{J}_i & \mathbf{0}_{7\times6} & \cdots & \mathbf{0}_{7\times6} & \mid & \mathbf{0}_{7\times5} & \cdots & \mathbf{0}_{7\times5} & \mathbf{J}_j & \mathbf{0}_{7\times5} & \cdots & \mathbf{0}_{7\times5} \end{pmatrix}\end{aligned}$$

where

$$\mathbf{J}_i = \frac{\partial \mathbf{e}_{ij}(\mathbf{X}_i \boxplus \mathbf{\Delta x}_i, \mathbf{M}_j)}{\partial \mathbf{\Delta x}_i} \tag{1}$$

$$\mathbf{J}_j = \frac{\partial \mathbf{e}_{ij}(\mathbf{X}_i, \mathbf{M}_j \boxplus \mathbf{\Delta x}_j)}{\partial \mathbf{\Delta x}_j} \tag{2}$$

Recalling Eq. (13) of the manuscript, we defined the error between predicted and actual measurement $\mathbf{e}_{ij}(\mathbf{X}_i, \mathbf{M}_j) = \hat{\mathbf{Z}}_{ij} - \mathbf{Z}_{ij}$ as

$$\mathbf{e}_{ij}(\mathbf{X}_i, \mathbf{M}_j) = \begin{pmatrix} \mathbf{e}_\mathbf{p} \\ \mathbf{e}_\mathbf{d} \\ \mathbf{e}_o \end{pmatrix} \tag{3}$$

where $\mathbf{e}_\mathbf{p}$, $\mathbf{e}_\mathbf{d}$ and $\mathbf{e}_o$ indicate the errors respectively between the origins, the directions and the orthogonality. Finally, we will use the following notation for the predicted and actual measurement:

$$\begin{aligned}\mathbf{Z}_{ij} &= \langle \mathbf{p}_{ij}, \mathbf{R}_{ij}, \mathbf{\Lambda}_{ij} \rangle \\ \hat{\mathbf{Z}}_{ij} &= \langle \hat{\mathbf{p}}_{ij}, \hat{\mathbf{R}}_{ij}, \hat{\mathbf{\Lambda}}_{ij} \rangle\end{aligned}$$

Indicating with $\mathbf{u}_x$ the unit vector $[1\ 0\ 0]^\top$, the Jacobian $\mathbf{J}_i$ in Eq. (1) is computed as

$$\mathbf{J}_i = \begin{pmatrix} \frac{\partial \mathbf{e}_\mathbf{p}}{\partial \mathbf{\Delta x}_i} \\ \\ \frac{\partial \mathbf{e}_\mathbf{d}}{\partial \mathbf{\Delta x}_i} \\ \\ \frac{\partial \mathbf{e}_o}{\partial \mathbf{\Delta x}_i} \end{pmatrix} \tag{4}$$

where

$$\begin{aligned}\frac{\partial \mathbf{e}_\mathbf{p}}{\partial \mathbf{\Delta x}_i} &= \begin{pmatrix} \hat{\mathbf{R}}_{ij}^\top & -\hat{\mathbf{R}}_{ij}^\top \lfloor \mathbf{R}_i \mathbf{p}_{ij} + \mathbf{t}_i \rfloor_\times \end{pmatrix} \\ \frac{\partial \mathbf{e}_\mathbf{d}}{\partial \mathbf{\Delta x}_i} &= \begin{pmatrix} \mathbf{0}_{3\times3} & -\lfloor \mathbf{R}_i \mathbf{R}_{ij} \mathbf{u}_x \rfloor_\times \end{pmatrix} \\ \frac{\partial \mathbf{e}_o}{\partial \mathbf{\Delta x}_i} &= \begin{pmatrix} \mathbf{0}_{1\times3} & \mathbf{u}_x^\top \mathbf{R}_i^\top \mathbf{R}_{ij}^\top \lfloor \hat{\mathbf{R}}_{ij} \mathbf{u}_x \rfloor_\times \end{pmatrix}.\end{aligned}$$

Here, $\lfloor \mathbf{v} \rfloor_\times$ denotes the skew-symmetric matrix built from vector $\mathbf{v}$.

Analogously - according to Eq. (2) - $\mathbf{J}_j$ is composed as follows:

$$\mathbf{J}_j = \begin{pmatrix} \frac{\partial \mathbf{e_p}}{\partial \boldsymbol{\Delta} \mathbf{x}_j} \\ \\ \frac{\partial \mathbf{e_d}}{\partial \boldsymbol{\Delta} \mathbf{x}_j} \\ \\ \frac{\partial \mathbf{e}_o}{\partial \boldsymbol{\Delta} \mathbf{x}_j}. \end{pmatrix} \tag{5}$$

In the remaining we will indicate with $\lfloor \mathbf{v} \rfloor_{\mathrm{F}}$ the clipped skew-symmetric matrix built from vector $\mathbf{v}$ as:

$$\lfloor \mathbf{v} \rfloor_{\mathrm{F}} = \lfloor \mathbf{v} \rfloor_{\times} \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Given these operators, Eq. (5) will expand in the following quantities:

$$\begin{aligned} \frac{\partial \mathbf{e_p}}{\partial \boldsymbol{\Delta} \mathbf{x}_j} &= \begin{pmatrix} -\hat{\mathbf{R}}_{ij}^{\top} & -\lfloor \hat{\mathbf{R}}_{ij}^{\top} \left(\mathbf{R}_i \mathbf{p}_{ij} + \mathbf{t}_i \right) - \hat{\mathbf{p}}_{ij} \rfloor_{\mathrm{F}} \end{pmatrix} \\ \frac{\partial \mathbf{e_d}}{\partial \boldsymbol{\Delta} \mathbf{x}_j} &= \begin{pmatrix} \mathbf{0}_{3\times 3} & \hat{\mathbf{R}}_{ij} \lfloor \mathbf{u}_x \rfloor_{\mathrm{F}} \end{pmatrix} \\ \frac{\partial \mathbf{e}_o}{\partial \boldsymbol{\Delta} \mathbf{x}_j} &= \begin{pmatrix} \mathbf{0}_{1\times 3} & -\mathbf{u}_x^{\top} \mathbf{R}_{ij}^{\top} \mathbf{R}_i^{\top} \hat{\mathbf{R}}_{ij} \lfloor \mathbf{u}_x \rfloor_{\mathrm{F}} \end{pmatrix}. \end{aligned}$$